

Introduction to Bridging Professional Development

Bringing *mathematical argumentation* to middle school classrooms through innovative, standards-based professional development



Mathematical Argumentation

- It's both a mathematical practice and a way to learn.
- We teach argumentation because students should have access to this most powerful mathematical practice.
- We teach *through* argumentation so that students can gain understanding of concepts.

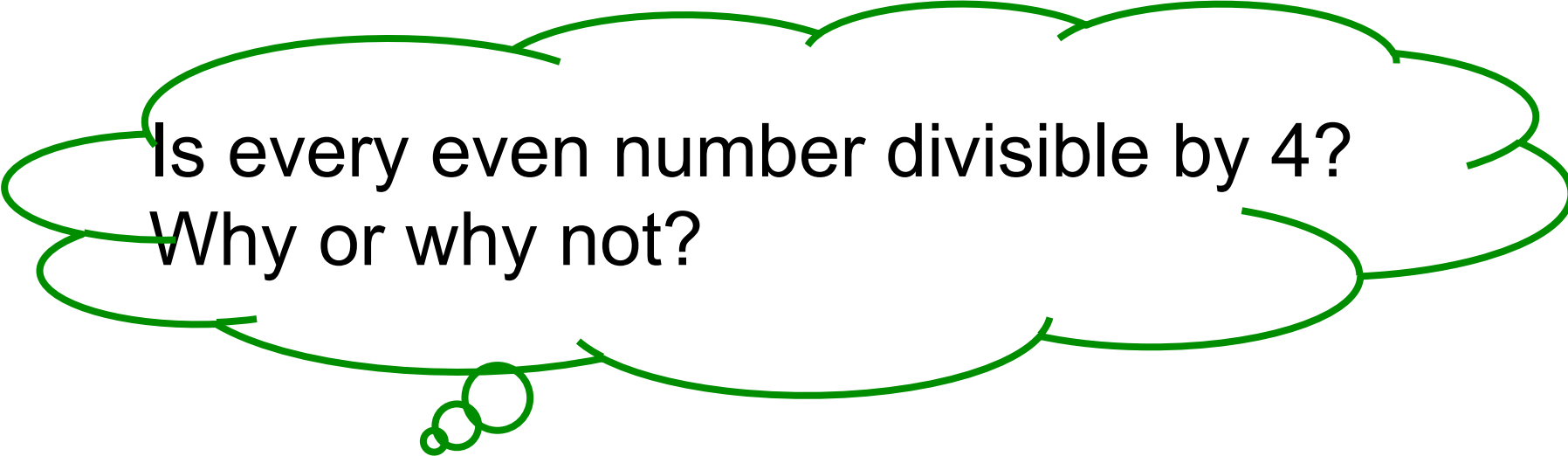
Common Core Math Practice #3

- “Create viable arguments and critique the reasoning of others.”
- One of four focused on in PARCC.
- Perhaps most important because justification can be used in almost any lesson.
- Do your students know what it means to justify?

Argumentation in three parts

- **Conjecturing**—making informed guesses about mathematical truth
- **Justifying**—creating a logical chain of statements to support or disprove a conjecture
- **Concluding**—deciding on the truth of a conjecture.

A first argument:



Is every even number divisible by 4?
Why or why not?

A first argument

Every even number is divisible by 4.
Because 4 is just 2 times 2,
and every even number is divisible by 2.

But what about 6? It's even
but 4 doesn't divide into it evenly.

Yeah, you're right. OK, let's see....
I think it's this way: Every number that
is divisible by 4 has to be even.

But how do you KNOW that?

Well, an even number divides in 2 evenly. If it's divisible by 4,
that has to be true, right?

OK, dividing into 2 evenly also means
"is divisible by 2." If you can divide it by 4, you can
divide it by 2

So now we know: Every number that is divisible by 4 is even.
But not every even number is divisible by 4.

Three Parts

Every even number is divisible by 4.

Conjecturing

Because 4 is just 2 times 2,
and every even number is divisible by 2.

But what about 6?
It's even but 4
doesn't divide into it evenly.

Yeah, you're right. OK, let's see....
I think it's this way: Every number that
is divisible by 4 has to be even.

But how do you KNOW that?

Well, an even number divides in 2 evenly.
If it's divisible by 4,
that has to be true, right? (etc)

Justifying

Concluding

So now we know: If 4 divides evenly
into a number, then the number has to be even.
But not all even numbers are divisible by 4.

Two prompts: not the same

Explain your reasoning

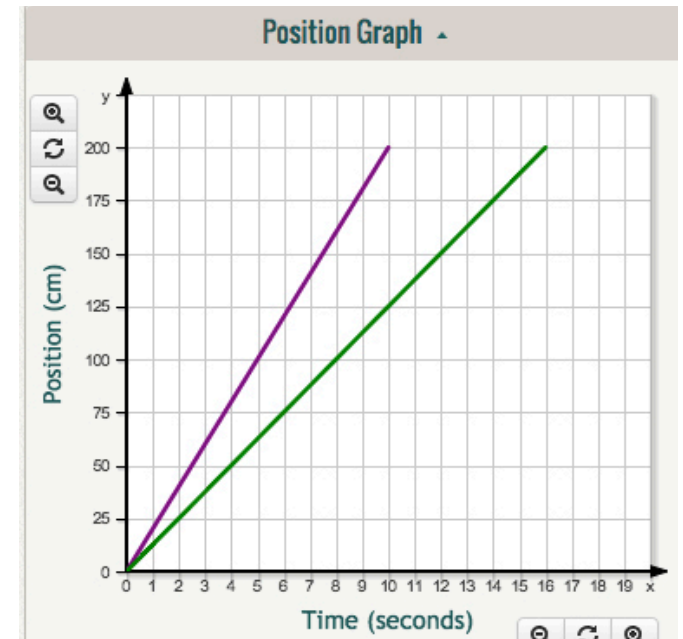
- Your reasoning is your own, a psychological process—can't say that's wrong.

Argumentation: How do you know it's true?

- Externalizes—need to communicate a compelling reason to someone else.
- Communication and standards of proof.

Tasks for Argumentation

- Problem solving
 - Create a trip with 3 segments that ends at 200 feet.
- Argumentation
 - Raj says that if a line is steeper than another, then it represents a faster motion. Is this always true?
- Modeling
 - The bus travels at 3 different speeds for city, country and highway. Design a route to get home in less than 3 hours.
- Precision
 - An object's time and position are noted at $(0,0)$, $(2,3)$, $(3,5)$ and $(10,14)$. What line best fits this data?



Teaching Moves for Argumentation

- Elicit conjectures: *What patterns do you see? Describe the patterns in a sentence.*
- Different forms of “why” questions:
 - *How do you know that?*
 - *How do we know it is true?*
 - *What makes you think so?*
 - *Show how you know.*
 - *Explain why this must be true.*
 - *What’s the mathematical reason it’s true?*
- *Explain to students what conjecturing, justifying and concluding are.*

Productive norms

Classroom Rights & Requirements

We are all in this together!

You have the right to...

- ...talk to a respectful audience.
- ...ask questions.
- ...have your ideas discussed, not you, personally.
- ...make mistakes.
-

You are required to...

- ...speak loudly enough for others to hear.
- ...really try to understand.
- ...give your own opinion on other people's ideas.
-

Norms and Agreements for Argumentation

- Make bold conjectures.
- It's OK to be wrong.
- Find out the mathematical truth together.
- Build off other people's ideas.

Zip, Zap, Zop: Improv Game for Norm Setting

- Stand in a circle. Each person throws an invisible ball to someone, saying “zip,” “zap,” or “zop” (one each, in that order).
- Keep going, in any order.
- Circus bow: what to do when you mess up.

Norm

- It's OK to be wrong. Celebrate mistakes!

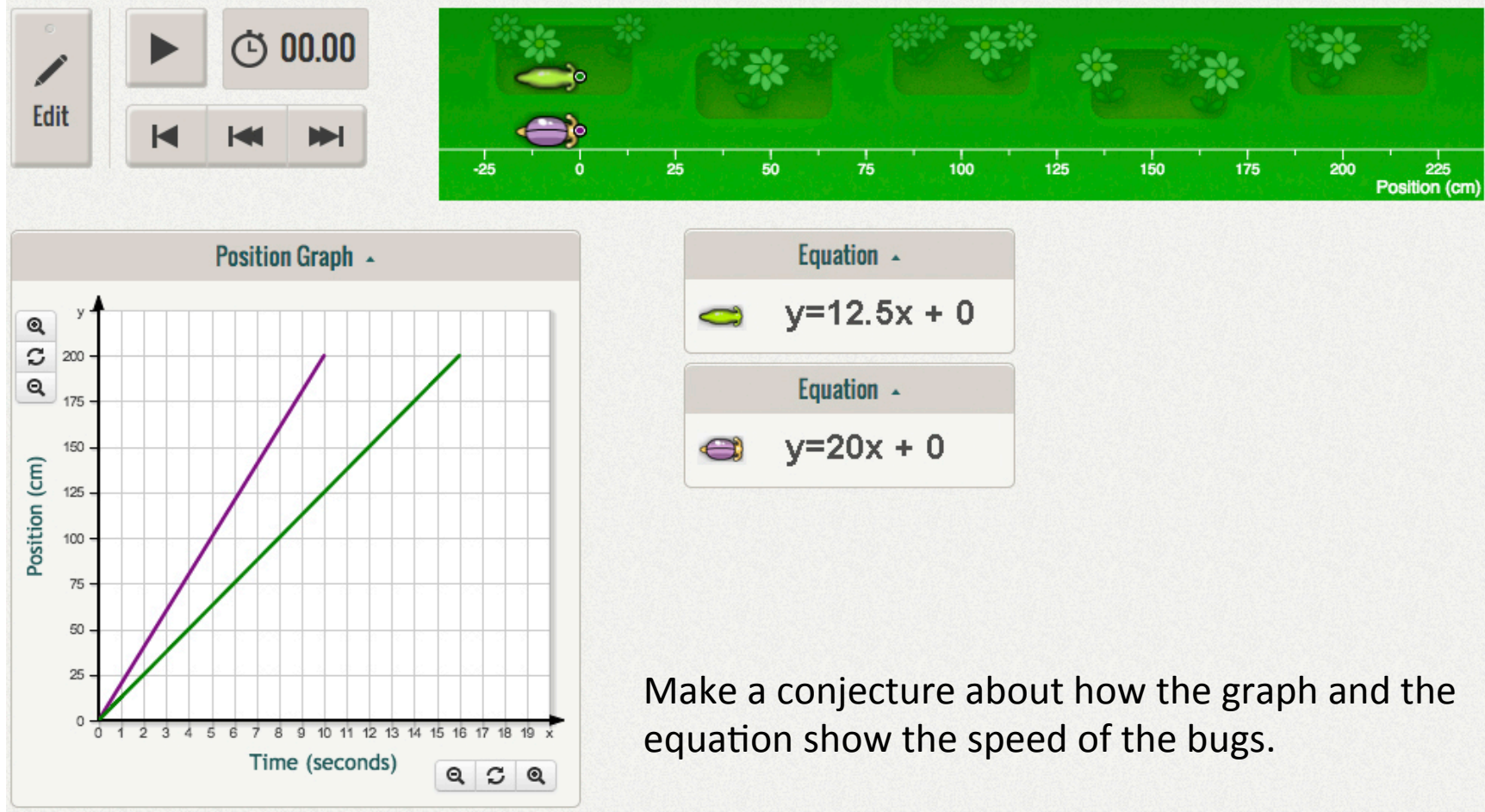
Gift Giving: Improv Game for Norm Setting

- Partners stand facing each other with a huge closet of unlimited gifts behind them.
- One player offers their partner a gift from the closet by handing them a gift wrapped in a box. This gift can be of any dimension, and the exchange gives an offer of size, weight, and/or shape.
- The receiver then opens the gift and names the present by thanking the giver (ex. "Thank you for this grapefruit.") as they pick up and handle the gift.
- The giver then responds with how they picked the gift and why they knew the receiver would enjoy it. The roles are then switched.

Norms

- Make bold conjectures
- Really try to understand and give your opinion.



Interactive Online Curriculum for Argumentation—Demo Lesson



Log in for our lesson

- Put instructions here. Participants do not have to log in, as we will do it whole group, but they can.

Conjecture Table from Teacher Guide

Conjecture	Justification	Conclusion
If its line is steeper than the other's, the bug will win.	<p>Counterexample:</p> <p>Even with a steeper graph, the bug may not catch up to another with a headstart.</p> 	False
If the graph of the bug starts higher (starting ahead), the bug will always win.	<p>Counterexample:</p> <p>Its graph starting higher doesn't guarantee that the bug will win. The other bug, if traveling faster, could catch up and win.</p> 	False
If one bug's graph starts at the same place as the other bug's, but is steeper, it will win.	The bug that starts ahead and moves faster will always win.	True
If the end points have the same y-coordinates but different x-coordinates, the one with the smallest x-coordinate wins the race.	The same y means that both travel the same distance. The smallest x means shortest time. So spending shortest time to travel the same distance means winning.	True

History and Effectiveness

- Development
 - Developed over 9 years in diverse settings including urban districts.
 - Current PD co-designed with DCPS staff and teachers.
- Past Results
 - In an impact study, Bridging students engaged in twice as much argumentation as “control” students.
 - In a study of four diverse classrooms, students learned both content and argumentation skills in discourse and writing.

Professional Development Opportunity: You are Invited!

- Summer workshop dates: August 11 – 14.
- School year follow up.
- You get paid at standard rates.
- You can fill out an application today.
- *Addresses TLF and DCPS curriculum framework.*



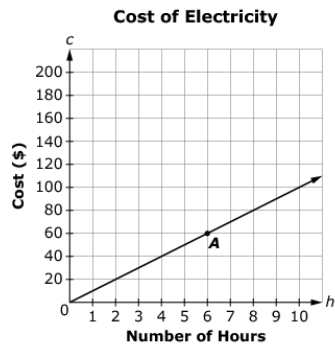
Bridging
Professional Development
for Mathematical Argumentation

promoting high-level mathematics for all students

Modifying a Problem

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This graph shows a proportional relationship between the number of hours (h) a business operates and the total cost (c) of electricity.



Select True or False for each statement about the graph.

	True	False
Point A represents the total cost of electricity when operating the business for 6 hours.	<input type="checkbox"/>	<input type="checkbox"/>
The total cost of electricity is \$8 when operating the business for 80 hours.	<input type="checkbox"/>	<input type="checkbox"/>
The total cost of electricity is \$10 when operating the business for 1 hour.	<input type="checkbox"/>	<input type="checkbox"/>

- Problem solving
- Argumentation
- Modeling
- Precision